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ABSTRACT

Problem solving is viewed as an increasing area of interest that focuses on the theory, research, and teaching of general cognitive skills which are applicable to broad problem classes. This document gives an overview of some important aspects of the theory and research in order to identify what general cognitive skills ought to be taught. Modern cognitive psychology, chiefly represented by Piaget, and artificial intellitence, represented by researchers such as Allen Newell and Herbert Simon are identified and examined. Piaget's theory is reviewed in some detail, with observations from his work extended to certain results from information processes. Teaching implications are drawn from this examination and a course entitled, "Patterns of Problem Solving," taught under the Department of Philosophy, St. Louis University, is examined. (MP)

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Problem Solving: An Overview of the Theory and a Course Attempting to Apply it

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I. Introduction

Problem solving is an increasing area of interest that focuses on the theory, research, and teaching of general cognitive skills which are applicable to broad problem classes. The importance of understanding and the possibility of teaching such cognitive skills goes without saying; life in general, and academic and professional training and practice in particular, is but a series of problems, solved or unsolved, of more or less consequence. D. Tuma puts it well saying,

> Problem solving is of ubiquitous importance whenever knowledge must be flexibly used (i.e., in any endeavor requiring more than the rote memorization or recall of information). Hence problem solving plays a central role in all the sciences and in most other fields. Furthermore, there is an increasing need to teach improved problem-solving skills to students who must be adequately prepared to cope with a world characterized by growing complexity, rapid change, and vastly expanding knowledge. (Tuma, 1980, p. ix)

An important and often cited contrast to the teaching of problem solving skills, hinted at here by Tuma, is the storage concept of learning which acts as the traditional paradigm for college and university curriculum. In adhering to this paradigm, higher education has perceived itself as existing principally to transmit a cultural heritage and to impart information. Students have traditionally been trained to know that, and those cases to know how by applying a limited set of procedures, algorithms, or formulas, applicable to highly specific and well-delimited problem situations.

The value of content learning is of course not disputed. Indeed, it is out of this content of facts and limited procedures as a basis, by induction and abstraction, that general cognitive skills are isolated, and it is with much and varied practice back upon this content that the correctness and propensity to use these procedures become 'second nature' enough to be called skills.

Factual learning, with its many narrow and unrelated problem solving algorithms, is the necessary nexus (the richer the better) out of which and by means of which general problem solving skills are acquired. However, the point here is simply that content learning by itself is one sided and incomplete. For, if one of the chief jobs of education is to give the student the tools to pursue successfully and efficiently academics themselves, and in part ultimately to prepare him or her for a successful life and career, then what must be acquired in addition to content are those cognitive meta-skills which are transferable to widely differing content situations. Transferability is to be understood here in the broad sense of not only the potential applicability to a wide variety of particular problems, but also the propensity, or readied inclination, on the part of the individual to use these skills.

From ancient times to the recent past, the establishing of such skills was put forth as one of the chief purposes of such required courses as Mathematics, Grammer, Logic, and Latin. This aspiration, however, was dampened for at least Logic and Latin by the celebrated studies of Thorndike made in the 1920's (Thorndike, 1924). Thorndike's research showed that, at least for certain specific modes of instruction, there was in such courses little transfer of knowledge and skills to new task situations. I shall have more to say on transferability later. Suffice it to say, that in the past the intent of teaching general thinking skills, successful or not, has played a significant role in curriculum design.

From the literature and this author's experience, a course designed to instill general problem solving skills is not only needed within a high school or college curriculum, but whatismore there is evidence that such a course can be successful, at least for some of these skills. I shall outline the content and success of such a course later.

As for need, studies have shown that at least 50 percent of a broad cross section of college students cannot perform, or perform only with partial success, tasks involving formal patterns of reasoning (Wason, 1969; McKinnon, 1971; Tower and Wheatley, 1971; Renner and Lawson, 1973). These formal reasoning patterns are those specified under Jean Piaget's theory of the four stages of human cognitive development, which is presently perhaps the most widely adopted theory underlying problem solving research and heuristics. The above statistic is alarming because the material and instruction that college students encounter assumes that these reasoning capacities are either already developed or that they develope automatically. The skills that do develope are often so 'content bound' that a change of subject matter renders the student impotent and requires a complete and ab initio re-learning of otherwise structurally similar methods and strategies.

In attempting to address this need, I shall in the following give first and overview of some important aspects of the theory and research as to what general cognitive skills ought to be taught, and then look at a practical attempt at applying these results. Taking theory first, substantial research in problem solving processesss has been carried out along two lines: modern cognitive psychology, chiefly represented by Piaget (Piaget, 1958; 1964; 1972), and information processing and artificial intelligence represented by researchers such as Allen Newell and Herbert Simon (Newell and Simon, 1958; 1962; 1965; 1972). We will examine Piaget's theory in some detail, and then extend these observations by certain results from information processing.

II. Theory: Relationships through Combinatorial and Proportional Reasoning

Under Piaget's theory the important step in acquiring the use of general cognitive skills, at the level we are interested in here, is the transition between what he calls the concrete operational stage (7-11 years) and the formal operational stage (11-? years). The concrete stage is so called because the mental operations at this level are "object bound". According to Piaget, concrete operations are internalized and reversible thought schemata which are based on the manipulation of classes, relations, quantities, and qualities of objects. They "...are called 'concrete' because they relate directly to objects and not yet to verbally stated hypotheses." (Piaget, 1958, p. 100) By contrast, at the next and final stage of development, the formal operational thinker is capable of reasoning verbally without reference to objects. This loosening from the concrete allows him to form hypotheses which generate possibilities not corresponding to any particular experience. More importantly for Piaget, the formal thinker is free to advance to a higher and more general perspective where he forms what are called second-order operations or relations. That is, at this stage there is the ability to operate on operations (concrete or higher , or alternatley to establish relations between relations (concrete or higher, e.g., to comprehend proportionalities. In another context, Piaget refers to this ability as reflective abstraction and argues that it is the very essence of mathematical and logical systems development (Piaget, 1970). I read Piaget with Philips (Philips, 1975, p. 128), that the ability to form these second-order operations, and hence to establish previously unknown but more generalizable systems of relations by means of analogy or proportion, constitutes one of the fundamental characteristics of the formal operational level. Philips gives as an example,

> A Concrete Operational child knows that the product of replicating a given object 3 times is 3 (not counting the model, of course), and he knows that if that set of 3 replicated 4 times, he'll have 12 of the objects. The adolescent has mastered those operations, but he can also place the resulting sets into relationship to each other: 12 is to 3 as 4 is to 1. To put it another way, 12/4 has the same <u>form</u> as 4/1. Before he can understand such a relationship, the adolescent must be able to operate formally on concrete operations. (p. 129)[1]

This ability to focus on the similarity of relations or systems of relations as <u>forms</u> or <u>structures</u>, independent of the <u>content</u> of the terms so related, makes for transferability and hence generalization which was not possible before. At the formal stage, operations are performed, or alternately, relations are established, on first order relations as they are expressed now in propositions rather than as they attach directly to the concrete reality. Formal operations, then, "are ways of transforming propositions about reality so that the relevant variables can be isolated and relations between them deduced." (Parson's introduction to Piaget and Inhelder, 1958, p. xviii) According to Piaget there are 16 such binary propositional operators.

Consequently, it is at this stage that the use of an implicit propositional logic develops, which is superimposed on the logic of classes and relations of the concrete operational level. This correlates directly with the development of proportional reasoning because "...proportions presuppose second degree operations, and the same may be said of propositional logic itself, since interpropositional operations are performed on statements whose intrapropositional content consists of class and relational operations." (Piaget and Inhelder, 1958, p. 254) Proportional reasoning is, nevertheless, the more fundamental. For, formal thinking makes itself known not only by the constant utilization of the sixteen binary propositional operations or some combination of them, but also

and more fundamentally, by the sporadic elaboration of what Piaget calls "operational schemata". The sixteen propositional operators, to function psychologically, presuppose an organization and integration that constitutes a structure or system, and this "system itself implies a set of potential transformations which may become manifest or remain latent depending on particular conditions." (Ibid., pp. 307-8) These transformations are the formal operational schemata, and are worked out spontaneously by the subject when they are needed to solve certain problems. Formal operational schemata have three common features: 1) they are general and susceptible of varied applications, as opposed to concepts in the narrow sense; 2) they are less discovered in objects than deduced or abstracted from the subject's own operational structures; and 3) they all show some relationship to the sturcture of logical thought. (Ibid., pp. 309-10) The eight formal operational schemata are as follows.

Piaget's Formal Operational Schemata

1. Combinatorial operations.
   The ability to systematically make up all possible color pairs given 5 or 6 cups containing various colored tokens, i.e., the ability to vary a single factor at a time, "all other things being equal," or alternately, the ability to isolate the relavent variable.
2. Proportions.
   $W/W' = L'/L$, W's and L's are weights and lengths of a balanced scale.
3. Coordination of two systems of reference.
   The movement of a snail left to right on a plank can be compensated for by a displacement of the plank from right to left.
4. The concept of mechanical equilibrium.
   Understanding equilibrium schema when the elements of equilibrium are no longer of the same nature, or when only one is visible.

The following schemata are derived simultaneously from one or two of the above four.

5. The notion of probability.
   Relationships between confirming and possible cases, calculated as a function of the combinations, permutations or arrangements compatible with the given element.
6. The notion of correlation.
   Simultaneous use of probability and proportionality, e.g., how is smoking related to lung cancer?

7. Multiplicative compensation.
   Directly related to the notion of proportion, a subject understands that what a volume gains or loses in one diminsion is compensated by what it loses or gains in the other two.
8. The forms of conservation which go beyond direct empirical verification. For example, conservation of energy or the principle of inertia, principles that must be deduced and verified from their implied consequences.

Using formal operations according to the above achemata, the formal thinker can be contrasted with the concrete thinker as follows.[2]

A subject using <u>concrete reasoning</u>:

C1. Needs reference to familiar actions, objects, and observable properties.

C2. Uses classification, conservation, serial ordering and one-to-one correspondence in relation to concrete items above.

C3. Needs step - by-step instructions in a lengthy procedure.

C4. Is not aware of his own reasoning, inconsistencies among various statements or contradictions with other known facts.

A subject using <u>formal reasoning</u>:

F1. Can reason with concepts, relationships, abstract properties, axioms and theories.

F2. Uses symbols to express ideas.

F3. Applies combinatorial, classification, conservation, serial ordering and proportional reasoning in these abstract modes of thought.

F4. Can plan a lengthy procedure to attain given overall goals and resources.

F5. Is aware of and critical of his own reasoning, and actively checks on the validity of his conclusions by appealing to other information.

Note that Piaget states explicitly that the last four operational schemata above derive simultaneously from one or two of the first four. Moreover, his analysis of the experimental content of 3) and 4) show that they involve the same simultaneous synthesis of <u>inversion</u> and <u>reciprocity</u> that constitutes proportional reasoning in general, though in 3) and 4) these operations are applied to particular contexts.[3] Proportional or metaphorical reasoning, then, is the logically more fundamental, and therefore constitutes together with the combinatorial operations of 1), the core out of which is built the more particularized operational schemata of the formal stage of development.

Proportionality is so central that it is often used as a test for determining the attainment of formal level reasoning (Karplus, 1977; Karplus, 1978); for as Piaget's research shows, in all areas (space, speed, chance, etc.) the notion of proportionality does not appear until the beginning of the formal stage (Piaget, 1958, p. 164).

More fundamentally, it is to be observed that combinatorial and proportional operations are simply systematic ways of establishing relationships among terms. This is obvious for proportionality, and, for combinatorial operations, they are "...exactly the same as selecting the cases where a single factor varies (the others being held constant) so as to isolate universal relationships from simple contingent conjunctions and above all so as to be able to discover necessary relationships between variables." (Ibid., p. 43) In short, if we take the standard definition of a problem as the disparity between an initial state and a desired goal state, problem solving is then the systematic establishment of a set of relations which bridges, by integrating into a single structure, the gap between the partial relational structures of the initial and goal states. A problem solution is a synthesis which brings the initial and goal states into a single system of known relationships. The cognitive operations that achieve this are at bottom, according to the theory, combinatorial and proportional reasoning delimited appropriately to the content of specific problem areas, as with the above operational schemata 3) thru 8). From the perspective of heuristics, it is those general strategies that delimit this reasoning to particular problems which must be sought next.

Before we investigate these strategies it is important also for purposes of instruction to stress a point implicit in the above account, i.e., the awareness by the successful problem solver of his own reasoning processes.

This self-awareness is particularly important for the ability to generalize from one problem situation to another; in other words, for transferability. Simon points out that we now know the necessity of two conditions for successful transfer (Tuma, 1980, p. 82). The first condition is that transfer from a problem A to a problem B requires that some processes or knowledge used in solving problem B be essentially identical to processess or knowledge involved in solving problem A. In line with the above theory, this implies that transfer is essentially a second order problem solving process which secures a proportionality between the structure of solved problem A and the corresponding structure of problem B. The proportionality makes for a systematic duplication within B of the first order procedures used within the solution of A. Proportionality so used is, of course, the implicit foundation of the power and success in teaching by example. The second condition, pertinent here, is that to transfer substantial skills acquired in solving a problem, learners need to be made explicitly aware of these skills, abstracted from their specific context. Again, in light of the above theory, this is but the other side of the coin to condition one. Awareness of a skill divested of content means the conscious command of a schema which as such is applicable to structually identical problems. Here, "Know Thyself" also leads to the self-corrective "Heal Thyself", as the self-awareness of poor problem solving skills leads to their improvement as well as their accurate and economic application. For these reasons, the importance is often stressed of having students practice themselves in reporting and analyzing <u>protocols</u>, or verbal descriptions, of their own problem solving procedures (Bartlett, 1978).

Protocol analysis also plays a central role in the field of artifical intelligence and information processing, and it is in the latter that we return to the topic of general strategies. Here, the verbal reports of problem solutions made by expert human problem solvers are analyzed, and then computer programs are written according to the "subroutines" used to achieve the solutions. The result is a formal representation of the problem solving process in the form of an algorithm capable of mechanically simulating human results. Among the algorithms successfully simulated, there is descernible the repeated use of certain overall strategies. Three such strategies are found most often and would appear to be the bases of other but more refined versions. I will summarize the basic three here (Larkin, 1979).[4] The first and principle procedure is called Means-ends Analysis. Means-ends analysis involves assessing the difference between knowledge constituting the initial problem state and knowledge necessary to make up the solution goal state. This assessment is used then to select an action that reduces the gap between the two states by mapping a net of relationships between them.[5] Note, then, that Means-ends analysis is but an explicit application, at a higher level, of the fundamental relational element focused on by the above theory. The initial and goal state of a problem are each made explicit and sorted out as a structure of interrelated terms, and then an action (e.g., combinatorial or proportional analysis) is produced to integrate the difference into a solved problem. A second example of a general strategy is a kind of planning which might best be called Idealization. It involves replacing the original problem with an abstract and simplified version retaining only certain central features. The abstracted problem is solved as above, and then its solution is used as a guide to solve the more complex original problem.

The third general strategy is Decomposition, or "Divide and Conquer". A currently unattainable goal state is replaced by an intermediate simpler subgoal. This subgoal is then achieved, reducing the gap to the original goal which is then taken up once again.

Notice that both Idealization and Decomposition are alternate simplifications of Means-ends analysis, and tell how problem solving procedures are to be applied. What procedures to use depend upon the subject matter and can be, say as in mathematics problems, highly intricate and sophisticated algorithms, yet the ability to extrapolate these procedures to a specific problem requires, principally, combinatorial and proportional reasoning. Moreover, the theory implies that these algorithms are themselves developed chiefly by successive additions and refinements by means of these latter fundamental modes of reasoning. Put in another way, proportional and combinatorial reasoning are, by the theory, the fundamental elements in the problem solving process in two ways. Using an analogy from formal logic; these modes of reasoning constitute both the formation processess involved in building the highly structured and particularized problem solving algorithms, as well as being the transformation process whereby the developed algorithm is applied analogically to a specific problem. Means-ends analysis, Idealization, and Decomposition are then meta-algorithms or strategies whereby specific algorithms are applied systematically and with economy. Therefore, problem solving, according to the theory, involves three levels of reasoning: 1) at bottom, proportional and combinatorial reasoning; 2) built up from these, algorithms for more particular problems and their application by 1)-type reasoning; and 3), heuristics and general strategies to systematically apply the appropriate 2)-type algorithm.

Teaching problem solving, which we take up presently, must somehow address and integrate these three levels of reasoning in the problem solving process.

Before going on, it is important to note that the term heuristics in 3) is meant to refer to those most general schemata that describe the overall problem solving process, and the context in which such strategies as those above (Means-ends analysis, etc.) are to be applied. Such schemata are generally some refined version of Polya's outline (Polya, 1945):

1. Understand the problem.
2. Find the connection between the data and the unknown.
3. Carry out a plan of solution.
4. Examine the solution.

For a more refined version see Alan Schoenfeld, 'Can Heuristics be Taught', (Lochhead, 1979). Such schemata trade their power for generality, and are by themselves of little practical value until integrated into a layered approach with more particularized strategies and procedures.

III. Teaching Problem Solving

Having attempted to integrate some aspects of theory, we must next ask how and even if problem solving skills can be effectively taught. That these skills can indeed be taught is evidenced by the resulting increased scores on various standardized tests of reasoning and problem solving abilities as reported by Bloom and Broder (1950), Bauman (1978), and Whimbey & Whimbey (1978).[6] The particular course I am most familiar with is one in which I assisted, entitled "Patterns of Problem Solving", and taught under the Department of Philosophy, St. Louis University. The course was researched and developed by Dr. Steven Bartlett who has taught it over the course of several semesters. "Patterns of Problem Solving", introduced in 1976, parallels in many details a UCLA course developed by M. Rubinstein,

and utilizes as its principle text one by the same name as the course, written also by Rubinstein (Rubinstein, 1975).[7] Unlike the UCLA program which has yet to generate empirical evidence of its effectiveness, Dr. Bartlett's course has produced some impressive test results. Using the California Test of Mental Maturity, Short Form 1963, Level 5 (mental age range common to Grade 12, college, and adult), administered as both a pre- and post-test to 17 students in the fall 1976, the following results were produced (reported in a Final Report of Work Supported by the Lilly Endowment):

> A total of 82.4% of the students showed some increase in their IQ scores.
>
> In terms of language IQ scores, 23.5% of the students demonstrated an increase of 11-19 points. 35.3% showed an increase of 8-19 points.
>
> In terms of non-language IQ scores, 29.4% of the students demonstrated an increase of 13-24 points. 47% showed an increase of 6-24 points.

The results are particularly impressive since the average initial total IQ was 127.94, and it is empirically well established that increases in IQ scores become progressively more difficult to produce the higher the pre-test IQ mean.

"Patterns" includes as its content selected material from Rubinstein, starting with general problem solving precepts which are then to be applied to problems within specific domains and amongst more particular methods. These domains include 1) reflections on language, propositional logic, Venn diagram and tree diagram modeling and classification. 2) Computer fundamentals, binary representation of information, and flow-charting. 3) Simple concepts of probability, independent and exclusive events, combinations, etc. 4) Types of models and analogical reasoning. And 5), Decision theory; models for decisions and behavior. The general precepts Rubinstein ask the student to apply throughout an abundance of problems are:

1. Get the total picture by clarifying what are the initial and goal states of the problem.
2. Use analogies and metaphors whenever possible.
3. Break the problem up into stable substructures.
4. Work backwards in an attempt to bridge the gap from initial to goal state.
5. Use models for simplification and clarification.
6. Withhold your judgement from premature courses of action.
7. Change problem representation to avoid restrictive mind sets.
8. Ask the right questions so as to stimulate useful associations.
9. Have the will to doubt, accept premises as tentative which makes for innovation and flexibility.
10. Talk so as to pry loose of constraints, and give time for incubation.

The first five appear as alternate ways of stating the results of the above theory overview, and the last five pertain to useful 'psychological' attitudes of ones own self-conscious problem solving.

In addition, "Patterns" includes practice periods in analogies and proportions, as well as the out-of-class assignments of a design/invention project, and an explicit written protocol analysis by the student of his own processes in solving particular problems. Required reading is J. Adams' Conceptual Blockbusting (Adams, 1974), focusing on overcoming blocks to, and techniques for, conceptualization. An abundance of problems are assigned and graded, and then gone over in the lab session once a week.

IV. Summary

The content of "Patterns" is in keeping with the survey of the tri-level theory above, and concerning it we have evidence that general problem solving skills can be taught. Nevertheless, as is often pointed out in the literature, the gap between problem solving theory and practice is still a wide one, and the few problem solving courses such as "Patterns" have developed more by art than by science. Part of the art is adapting specifics to the level and background of the students involved.

No matter the level, however, I think the above theory and the content of the successful courses recommend to us the following general guides.

First and foremost, students must be practiced in the ability to "see", and the propensity to look for, relationships. The central importance of this ability cannot be over stated: it is fundamental to reasoning and intelligence in general. Whimbey goes so far as to define intelligence as "paying careful skilled attention to the analysis of relations." (Whimbey & Whimbey, 1978, p. 119) One way to do this is by combinatorial resoning as found, for example, in the combinations and permutations of simple probability theory. An additional and more direct way is by practice in proportionalities, metaphors, and analogies. In the words of William James, "...the faculty of perceiving analogies is the best indication of genius." Or, as Kaput states, "Metaphorizing is a central process by which we transfer meaning. We regard the creation and application of metaphors as an irreducible element of all human thought at all levels." (Lochhead & Clement, 1979, p. 294) One possible way to practice seeing relations using analogies would be to use as a text one of the commercially available programmed workbooks used as preparation for the Millers Analogies Test.

Building upon this, there could next be exercies in classifying and ordering as more specific ways of relating. Venn diagrams, tree diagrams, and flowcharts are appropriate here, and are easily applicable to any content background.

Practice in relating and metaphor must, of course, be carried further, for instance between the structure of a solved problem given as an example paradigm and a problem that the student must solve.

This brings us to the second level of the problem solving process involving particularized algorithms of limited scope. The difficulty here, especially in the developmental curriculum, is the choice of appropriate areas to build these algorithms up out of without presupposing too much background in specific subjects. Simple logic appears most appropriate, and possibly elementary probability, decision theory, and economics. Also fertile ground would be elementary arithematic, geometry and algebra, and possibly elementary physical science. The demand for an artful instructor is evident here.

Focusing a moment on logic, from my own experience the teaching of some basic logic would be of considerable value to most students. What I am thinking of here is not the symbolic or axiomatic approach often found in logic texts, but rather the teaching of some common valid modes of inference (e.g., Modus ponens, Modus tollens, etc.), and the recognition of common fallacies (e.g., Affirming the consequent, Ad Hominem, etc.) as they operate at the level of everyday natural language. Clarifying the distinctions between deduction and induction, validity and truth, and, matter and form, makes for such practical insights as that an argument can be valid ('reasonable') and yet can perfectly well have a false conclusion, or the inverse. Also, logic appears to me to be an excellent means of overcoming one of the 'habitual' modes of thinking of the low-aptitude student as described by Bloom and Broder (Bloom and Broder, 1950), i.e., one-shot and insight oriented thinking based upon a cursory evaluation and intuition, rather than extended, sequential analysis and reasoning. Logic would also help reduce another observed tendency, that of subjective opinions intruding in matters requiring purely logical considerations.

Finally, there should be practice in clarifying problems, rendering them well defined, i.e., what exactly is the problem; what system of relationships constitutes the given and what relationships are necessary to a solution. This establishes the prerequisite basis for the third level of the problem solving process involving exercises in strategies such as Means-ends analysis, Idealization, and Decomposition, and is the first step in the application of any overall heuristic schemata such as Polya's. Further, it is of the essence that a student have an explicit working overview of the problem solving process which implies the ability to adjust, when necessary, to any one of its three levels: searching for fundamental relationships, applying specific algorithms, and proceeding according to an overall heuristic and its sub-strategies. Practical experience dictates that in the actual classroom this will require the constant practice and the forced awareness of a small number of algorithms applied 'by the numbers' according to the sequence of the adopted heuristic.

Notes

1. Philips points out that not only do quantitative relationships become available at this time, e.g., 12:3::4:1, but also "comprehension and appreciation of metaphore, of irony and satire, of proverbs and parables, of analogies of all kinds." (p. 132)

2. From Module 9 of the Science Teaching and the Development of Reasoning workshop materials, Lawrence Hall of Science, Berkeley, Cal. 94720.

3. See (Piaget, 1958), pp. 176-80, 207, 314-17.

4. Newell gives a list of nine such strategies (Newell, 1980). However, those of the nine that are systematic (as opposed to trial and error groping) would appear to be but versions of the three we have listed.

5. For a detailed exposition of the protocols of two experts used in solving a problem using Means-ends analysis, see Simon, D. and Simon, H. in (Lochhead and Clement, 1979), pp. 119-132.

6. For a brief list of some of the more prominent universities which offer courses and interdisciplinary programs in problem solving see (Lochhead and Clement, 1979), pp. 2-3.

7. For a synopsis of Rubinstein's UCLA course and a more elaborate description of Bartlett's results see Rubinstein, A decade of experience in teaching an interdisciplinary problem-solving course, in (Tuma, 1980), pp. 25-38.

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